Math Virtual Learning

Calculus AB

Differential Equations

April 13, 2020

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Lesson: April 13, 2020

Objective/Learning Target:

Students will verify that an equation is a solution to a differential equation

Warm-Up:

Watch Video: Solving Differential Equations

Read Article: Solving Differential Equations

Examples:

EXAMPLE 1 Solving a Differential Equation

Solve the differential equation $y' = 2x/y$.

Solution

$y' = \dfrac{2x}{y}$	Write original equation.
$yy' = 2x$	Multiply both sides by y.
$\int yy'\,dx = \int 2x\,dx$	Integrate with respect to x.
$\int y\,dy = \int 2x\,dx$	$dy = y'\,dx$
$\frac{1}{2}y^2 = x^2 + C_1$	Apply Power Rule.
$y^2 - 2x^2 = C$	Rewrite, letting $C = 2C_1$.

So, the general solution is given by

$$y^2 - 2x^2 = C.$$

Examples:

THEOREM 6.1 Exponential Growth and Decay Model

If y is a differentiable function of t such that $y > 0$ and $y' = ky$, for some constant k, then

$$y = Ce^{kt}.$$

C is the **initial value** of y, and k is the **proportionality constant. Exponential growth** occurs when $k > 0$, and **exponential decay** occurs when $k < 0$.

EXAMPLE 2 Using an Exponential Growth Model

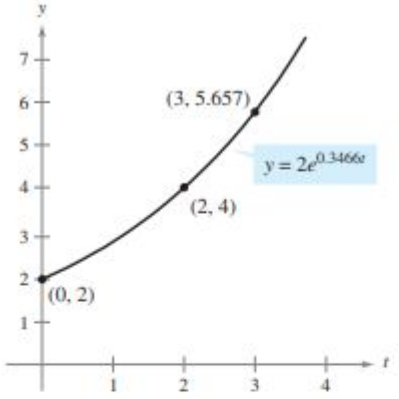
The rate of change of y is proportional to y. When $t = 0$, $y = 2$. When $t = 2$, $y = 4$. What is the value of y when $t = 3$?

Solution Because $y' = ky$, you know that y and t are related by the equation $y = Ce^{kt}$. You can find the values of the constants C and k by applying the initial conditions.

$$2 = Ce^0 \quad \Rightarrow \quad C = 2 \qquad \text{When } t = 0,\ y = 2.$$

$$4 = 2e^{2k} \quad \Rightarrow \quad k = \frac{1}{2}\ln 2 \approx 0.3466 \qquad \text{When } t = 2,\ y = 4.$$

So, the model is $y \approx 2e^{0.3466t}$. When $t = 3$, the value of y is $2e^{0.3466(3)} \approx 5.657$ (see Figure 6.8).



If the rate of change of y is proportional to y, then y follows an exponential model.

Figure 6.8

Practice:

1) Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram?

2) Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the fourth day. Approximately how many flies were in the original population?

Answer Key:

Once you have completed the problems, check your answers here.

1) Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram?

Solution Let y represent the mass (in grams) of the plutonium. Because the rate of decay is proportional to y, you know that

$$y = Ce^{kt}$$

where t is the time in years. To find the values of the constants C and k, apply the initial conditions. Using the fact that $y = 10$ when $t = 0$, you can write

$$10 = Ce^{k(0)} = Ce^{0}$$

which implies that $C = 10$. Next, using the fact that $y = 5$ when $t = 24{,}100$, you can write

$$5 = 10e^{k(24{,}100)}$$

$$\frac{1}{2} = e^{24{,}100k}$$

$$\frac{1}{24{,}100}\ln\frac{1}{2} = k$$

$$-0.000028761 \approx k.$$

So, the model is

$$y = 10e^{-0.000028761t}.$$ Half-life model

To find the time it would take for 10 grams to decay to 1 gram, you can solve for t in the equation

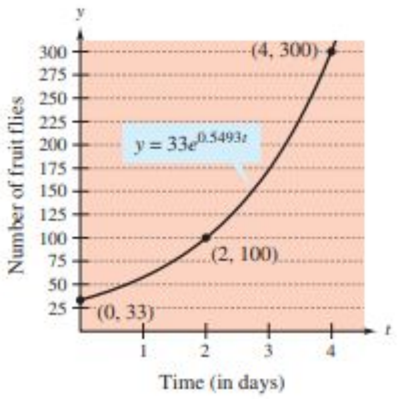
$$1 = 10e^{-0.000028761t}.$$

The solution is approximately 80,059 years.

Answer Key:

Once you have completed the problems, check your answers here.

2) Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the fourth day. Approximately how many flies were in the original population?



Solution Let $y = Ce^{kt}$ be the number of flies at time t, where t is measured in days. Because $y = 100$ when $t = 2$ and $y = 300$ when $t = 4$, you can write

$$100 = Ce^{2k} \quad \text{and} \quad 300 = Ce^{4k}.$$

From the first equation, you know that $C = 100e^{-2k}$. Substituting this value into the second equation produces the following.

$$300 = 100e^{-2k}e^{4k}$$
$$300 = 100e^{2k}$$
$$\ln 3 = 2k$$
$$\frac{1}{2}\ln 3 = k$$
$$0.5493 \approx k$$

So, the exponential growth model is

$$y = Ce^{0.5493t}.$$

To solve for C, reapply the condition $y = 100$ when $t = 2$ and obtain

$$100 = Ce^{0.5493(2)}$$
$$C = 100e^{-1.0986} \approx 33.$$

So, the original population (when $t = 0$) consisted of approximately $y = C = 33$ flies, as shown in Figure 6.9.

Additional Practice:

In your Calculus book read section 6.2 and complete problems 1, 5, 21, 23, and 41 on page 418

Extra Practice with Answers